Conclusions

The theory developed uses base flow concepts as they apply to either the small-bleed or the large-bleed regime. Experimental performance data taken from cold-flow and simulated hot-flow tests show that the theoretical model is capable of predicting both qualitatively and quantitatively the influence of the major design and operational variables. The results indicate that the use of base bleed provides good performance gains over that obtained from the basic nozzle thus indicating that the aerobell provides a method for uprating the performance of current bell nozzle thrust chambers with a minimum of development effort.

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A Hazards Model for Exploding Solid-Propellant Rockets

J. C. McMunn*
TRW Systems, Redondo Beach, Calif.

AND

Jon D. Collins†
J. H. Wiggins Company Inc., Palos Verdes Estates, Calif.

AND

Billings Brown‡
Hercules Powder Company, Magna, Utah

A model is developed for predicting in a probabilistic sense the number of casualties or fatalities resulting from an exploding solid-propellant rocket. A casualty expectation equation is developed in terms of the degree of population protection, the missile impact probability in any populated area, the casualty area of the hardware and firebrand debris and the land area surrounding the impact point. The casualty area is a function of impacting propellant weight and therefore varies with flight time. These expressions are based as much as possible on data from actual flight failures and tests of both composite and double-base propellants. A numerical example is presented for a hypothetical missile; and the accuracy of the casualty expectation is estimated by computing the standard deviation. The primary use of this model is to control the planning of missile flights in order to protect people.

Introduction

SOLID-PROPELLANT rockets have propellants that are often classified as high explosives for purposes of transportation and handling. Special precautions must be taken during launch and flight to insure adequate protection to humans; among these include onboard destruct devices, flight corridors away from populated areas, and evacuation near the launch site. Sometimes a malfunctioning rocket is uncontrollable resulting in an erratic flight that breaks the rocket into several pieces; at the same time the Range Safety Officer initiates the destruct device which punctures each stage case igniting the propellant. The ignited stages explode upon ground impact producing an overpressure region and many firebrands which are thrown out to large distances. Thus, even though rocket flight is terminated, a hazard is present which is particularly acute early in flight when most of the propellant is unburned. At later flight times the main hazard is from rocket hardware. The objective of this paper is to examine the nature and the extent of the explosion from impacting solid-propellant rockets and to assess the resulting hazards. Hazards are defined only in terms of risk to human life.

In accomplishing this objective a model is developed which quantifies risk, a term which is generally treated only qualitatively. A conservative approach is taken so that computed

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* Head, Structural Dynamics Section, Applied Mechanics Laboratory.

† Vice President. Member AIAA.

‡ Senior Technical Specialist, Advanced Products Engineering. Member AIAA.

casualty expectations will more than likely be large. An acceptable casualty expectation level must be based on relative values. A casualty expectation of 10^{-7}, for example, means very little until weighed against a casualty risk of 10^{-6} for a person driving an automobile 10 miles. The acceptable risks will not be pursued further, but it is necessary to point out that the results of this paper are addressed to this type of consideration.

Factors that must be included to perform a numerical hazards study and evaluate casualty expectations can be broadly grouped into two categories: those that involve characteristics of the particular missile and those that involve the environment. The missile category includes quantity and type of propellant in each stage, missile range capability, flight failure probabilities, and missile break-up characteristics due to aerodynamic forces and due to destruct charges. The environmental category includes launch site location and nominal launch azimuth, distribution of the population near the flight path, and wind velocity and direction. This paper is restricted to the factors involving the missile and more specifically those associated with the breaking-up of the missile and the consequent impacting propellant and inert debris on the ground.

Casualty Expectation

Consider an area of land A_T with human inhabitants that have varying degrees of protective cover from falling missile debris and blast overpressures. At any given time of the day the population can be categorized according to protection. For example, the population might be equally divided between being in the open, in frame dwellings, and in masonry structures.

Assume n levels of protection within A_T and denote the areas occupied by these levels by A_i, $i = 1, \ldots, n$. Evidently,

$$A_T = \sum_{i=1}^{n} A_i$$

Let N_i be the number of people in protective level A_i and let $P_{I_{ij}}$ be the probability that the jth piece of debris will land in A_i.

The jth piece of debris will have n casualty areas $A_{c_{ij}}$, one for each level of protection. The term casualty area describes that area surrounding the impact of a piece of debris whose bounds are the 50% probability point that a casualty will occur. Thus, if an explosion were to occur upon impact, a radius could be defined within which the probability of casualty to people is more than 50% and beyond which is less than 50%.

We can express the casualties expected in A_i from the jth piece by

$$E_{c_{ij}} = P_{I_{ij}} N_i A_{c_{ij}} / A_i \tag{1}$$

Equation (1) is derived on the assumption that the population is uniformly distributed in A_i but is equally valid for completely random positioning within the area.[1] Next, by summing over all pieces and all protection categories, we obtain the total casualty expectation

$$E_c = \sum_{i=1}^{n} \sum_{j=1}^{m} P_{I_{ij}} N_i A_{c_{ij}} / A_i \tag{2}$$

If A_T is small, the impact probability will be nearly uniform over A_T. For this case, the probability of the jth piece impacting in A_i will be $(A_i/A_T)P_{I_j}$. Furthermore, if the impact distribution for each piece is the same then $P_{I_j} = P_I$. Equation (2) can now be further simplified by normalizing N_i and $A_{c_{ij}}$. Let $n_i = N_i/N$, i.e., let n_i be the fraction of the total population in protection category i; and let $a_{ij} = A_{c_{ij}}/A_c$, i.e., let a_{ij} be the ratio of the casualty area in category i to a reference casualty area, A_c. Then,

$$E_c = \frac{P_I N A_c}{A_T} \sum_{i=1}^{n} \sum_{j=1}^{m} n_i a_{ij} \tag{3}$$

The choice of the size of A_T, for the assumption $P_I = P_{I_{ij}}$ to remain valid depends largely on its location relative to the launch site and the nominal trajectory path. Near the launch site it may be necessary to consider individual buildings or to subdivide a small village. At large distances from the launch site the impact probability may be uniform over a small country and if the population is considered distributed homogenously in the country, then A_T may be taken as the country.

Solid-Propellant Explosion Characteristics

The problem of predicting accurately the casualty area of an exploding solid-propellant rocket is complex. There is no theory available that can predict the "explosiveness" of any solid propellant under all impact conditions. Even if the pressure-time history was known during the explosion, the fracture mechanics of the propellant and stage case are not well enough understood to predict the number and sizes of fragments. Therefore, the only way to realistically assess the hazards upon impact is to survey existing debris maps, motion pictures of failures, overpressure data, and donor initiated solid-propellant tests. An attempt must then be made to correlate the data and establish trends of overpressure and fragment-throw distance with quantity and type of propellant and donor energy or impact conditions.

For purposes of handling and storage, solid propellants are commonly divided into two classifications—either military class 2 (fire hazard only) or class 7 (mass detonating hazard). Most solid propellants having reactive binders containing nitro or nitrato groups (nitroglycerin or other compounds together with nitrocellulose) are class 7, whereas those having nonreactive binders such as polybutadiene or polyurethane are class 2. However, as propellant energy levels have increased, the distinction between the two families has diminished. Some composite propellants are now class 7 whereas some double base are class 2. Many multistage solid-propellant rockets have both class 2 and 7 propellants. For example, the Minuteman missile has two class 2 stages and one class 7 and Scout has three class 2 stages and one class 7. Before discussing propellant tests it is appropriate first to define several terms.

A *detonation* is distinguished from other explosions in that the detonation wave moves through the propellant at a velocity greater than sonic velocity. Energy is transferred to the unreacted material by hydrodynamic shock heating. The detonation velocity is determined by several factors. Some materials exhibit two distinct stable velocities that may differ by as much as a factor of five. A detonation completely consumes the propellant so there are no firebrands or burning propellant debris and the chemical energy is transformed into a hemispherical shock front which propagates outwards. A *deflagration* is an explosion in which the decomposition wave front moves through the material at a velocity less than sonic velocity but still in excess of the velocity of sound in air. Energy is transferred to the unburned material by conduction and convection. A deflagration results in a much smaller overpressure and numerous firebrands that are propelled to large distances. Even though the deflagration wave in the material is subsonic, a relatively strong shock can exist in the atmosphere.

There are explosions that are all "shades of gray" between gentle burning and detonation. Some tests have shown a fading detonation in which one end of the propellant exhibits a true detonation, the detonation decaying and the remaining propellant charge deflagrating or even being extinguished.

The *critical diameter* is the propellant diameter necessary to sustain a stable, steady-state detonation. The critical diameter depends on many factors, some of which will be discussed. Below the critical diameter, the rate of energy loss by lateral dispersion exceeds energy release, and the wave front velocity decays. The critical diameters of double base propellants do not exceed 4 in. and are normally less than 1 in.

Review of Small-Scale Laboratory Tests

Let us first review the small-scale laboratory tests that have been performed to establish propellant hazard classification. The Naval Ordnance Laboratory Card Gap Test has been used for years to establish the minimum energy required to initiate a detonation. However, this test is only performed on small specimens and therefore does not recognize the fact that propellants have a critical diameter. The critical diameter of composite propellants was until recently thought to be very large, possibly as large as 660 in. Several investigators stated that it did not exist.[2] Recent results of Project SOPHY (Solid Propellant Hazards Program conducted by the Aerojet-General Corporation)[3] have demonstrated that the critical diameter of PBAN (polybutadiene, acrylic acid, acrylonitrile copolymer) composite propellant is 64 in. The SOPHY team actually detonated a 72-in. cylindrical charge using a conical shaped TNT charge as an axial end donor while a 60-in. charge failed to detonate. SOPHY results also showed that detonability depends on material porosity with increasing porosity reducing the critical diameter. The tests indicate that the pressure necessary to detonate a PBAN composite propellant is 30–40 kbars and might be as low as 8–10 kbars. The 8–10 kbar figure is based on extrapolation. This pressure might be generated by a falling missile stage under ideal impact with a terminal velocity of around 500 fps.

In the witness plate test, a steel plate is placed at one end of a tube that is charged with propellant. If a hole is sheared in the plate, the propellant detonated; if the plate is bent there was no detonation by definition. This test does not recognize propellant hazards since a deflagrating charge that does not shear a witness plate can throw burning propellant to large distances which could easily result in more casualties than a detonating charge.

Another important parameter influencing detonability that has not been studied sufficiently is donor pulse width. Here donor is used to mean an explosive charge or the pulse generated on impact. There is evidence[4] that detonability depends on pulse width. The pulse width may be much narrower for a high-explosive donor than for a ground impact shock. For the aforementioned reasons extrapolation of small-scale laboratory tests to predict large solid-propellant stage hazards seems risky and may lead to false conclusions. In addition, no test or classification recognizes the hazard associated with the thousands of firebrands thrown in all directions to large distances when a missile impacts after failure.

Large-Scale Tests and Missile Failures

In the past decade there have been many large-scale donor initiated solid-propellant tests conducted at government and industrial test facilities. In addition, there are still and motion pictures of flight failures near launch sites and debris maps of some of these failures. The donor initiated tests usually do not simulate a range safety destruct device or a ground impact condition, so care must be taken when these data are used in attempting to predict in-flight hazards. Flight failure debris maps usually show only where major hardware and case fragments are thrown and in no way reveal the firebrands of burning propellant or the magnitude of overpressure. Usually flight failures are in uninhabited areas, so there is little structural damage to use as a guide to the damage that might result in a populated area.

The ballistic coefficient of most large solid-propellant stages ranges from 100 to 300 psf. For this ballistic coefficient range the terminal velocity for a falling stage will not exceed around 500 fps. There have been two sled propelled impact tests of large missile stages. One sled test attempted to duplicate a powered impact of a 120-in.-diam composite propellant by impacting it into a concrete wall at 667 fps. It failed to detonate. This particular stage was from Titan III-C and based on pressure measurements had an explosion which was equivalent to 8.5% TNT.[5] TNT equivalency is based on weight, i.e., a pound of propellant with a 200% TNT equivalency yields the same overpressure as 2 lb of TNT. Another sled impact test was conducted on the Minuteman missile. In this test, the three stages were separated by 7.5 ft and mounted on wheeled dollies on a sled track. The first stage was ignited and used to propel the missile into an impaler at 60 fps. The missile exploded yielding 1% TNT equivalence. Burning propellant was thrown to 1200 ft.[6]

There have been a number of high-explosive donor initiated tests conducted at the China Lake Naval Ordnance Test Station. Many of these tests were conducted to assess the propellant hazard for the Armed Services Safety Explosive Board. A typical test consisted of placing a small charge on the case or dome of a single stage and detonating the donor. Pressures were measured from the resulting explosion to determine TNT equivalence, and for some tests, maps were made showing hardware, case and unburned propellant distribution. High-speed movies were taken and can be used to count the number of burning propellant pieces thrown out from deflagrations. A series of these tests was reported by Weals and Wilson.[7] These tests did not attempt to reproduce impact conditions from a falling stage or the range safety destruct mechanism. For example, a small donor placed on the case can detonate a class 7 stage whereas the same stage impacting at several hundred fps will deflagrate yielding only a small TNT equivalence.

There have been a number of flight failures with solid-propellant rockets. There are still and motion pictures of most flight failures near launch sites, and in most cases, whether the stage was destroyed by the Range Safety Officer or self-destroyed, the missile exploded with a display that resembled fireworks with a shower of burning propellant thrown out to several thousand feet. A review of many of these flight failures shows that neither composite nor double base falling stages detonated upon ground impact. These flight failures resulted in deflagrations yielding a relatively small overpressure. Usually it was estimated to be under 20%, but with a large number of burning propellant chunks. Oftentimes the missile was ignited in-flight by the destruct device and fell burning until it impacted and exploded. The number of burning pieces varied with missile size and time from launch, but in some cases there were several thousand. The debris maps were made several days after the failure and only charred case and hardware fragments remained with little unburned propellant.

Explosion Hazards Model

There are three important effects of exploding solid-propellant missiles that must be included in the calculation of a casualty expectation. The first is the effect of the overpressure upon the people in the open near the explosion. The second is the effect of overpressure on houses and lightly constructed buildings near the explosion. Here overpressures can collapse the structures, trapping occupants in the debris, and then the firebrands can ignite the buildings and burn the trapped occupants. The third effect is beyond the overpressure region where there are missiles of burning propellant and inert debris. In this area, people in the open can be

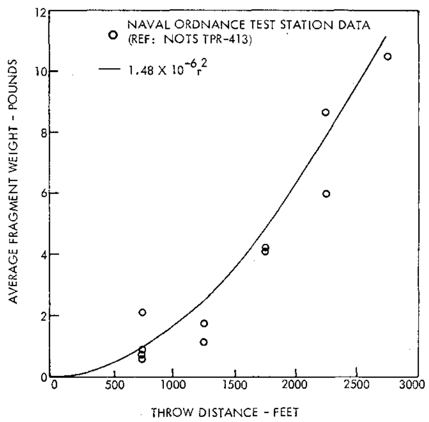

Fig. 1 Average fragment weight at radial locations.

killed or injured by direct hit or by a bouncing piece of burning propellant, and occupants of a house can be hit by a collapsing roof or by debris penetrating the roof.

After reviewing missile explosion characteristics, it becomes apparent that it is impossible to account for several thousand individual pieces of burning propellant and, therefore, it seems appropriate to introduce averages for size, weight and number of pieces. Motion pictures and debris data can be used to establish these averages.

Model Assumptions

The preceding discussion is used as a framework for stating the following assumptions which serve to guide the analytical model.

1) Destruct devices used to halt powered flight will not detonate a class 7 propellant.

2) A stage never impacts with a velocity sufficient to initiate a detonation. Hence, class 2 and 7 propellants are indistinguishable in this analysis. Both deflagrate upon impact resulting in overpressure, firebrands and inert fragments.

3) A TNT equivalence per pound is assigned to each stage. The TNT equivalence is based on actual flight failures. The overpressure-distance relation is based on tests by Kingery.[8]

4) Propellant not converted into the shock wave is ignited and thrown outward as firebrands.

5) The firebrand distribution is assumed to be axisymmetric. The firebrand average size and radial distribution can be estimated from pictures or from donor test debris maps. For the model developed the distribution is based on donor tests.

6) All firebrand pieces bounce upon ground impact due to their resiliency. Each piece is assigned a 2 ft^2 casualty area to account for human dimensions. Heat radiation from the deflagration is assumed negligible.

Computation of the Casualty Area

The casualty areas of hardware and burning propellant debris are defined in the same manner as in previous range safety studies.[9] Casualty area as discussed earlier is that portion of occupied ground upon which a person would suffer injury from a direct hit by falling or bouncing missile debris, or from the overpressure generated by an exploding stage.

It is not possible to accurately describe the shape and weight of all pieces of debris. Certain pieces of missile hardware usually remain intact and, for these pieces, the casualty area can be computed from missile drawings. These items iuclude electronic packages, batteries, nozzles, cables, raceways and other similarly small items. The burning propellant pieces are of many shapes. For these pieces we will assume they are all rectangular parallepipeds.

The shape and area of a piece of debris does not account for the size of a person which must be included in the casualty expectation. The effect of a person on the casualty area of a single piece can be accounted for by increasing the dimensions of the pieces so that the increased area is at least equal to the area occupied by a standing person which is considered to be 2 ft^2. Therefore, the minimum casualty area is 2 ft^2. If the piece is large, the dimension of a person still has an effect. For example, a piece of debris with plan view dimension of 6 ft $\times$ 6 ft would cause a person to become a casualty when that person is located anywhere within the 36 ft^2 area. Furthermore, a person standing within approximately 1 ft of the perimeter would also be a casualty, since the person would be hit on at least part of the body.

Hence the proposed model for the casualty area due to debris is computed by first converting the plan view area of the piece to a circular piece with equal area and then adding 1 ft to the radius to account for human size. The modified radius is then used to compute the area of a circle which is the casualty area. Expressing this statement in a formula we obtain

$$A_c = [(A_{\text{proj}})^{1/2} + (2)^{1/2}]^2 \tag{4}$$

This expression is used to compute the casualty area for each debris piece for people in the open (category 1). This formula is normally conservative because it overestimates casualty area in areas of high debris density.

Overpressure Casualties

Let W_p be the impacting propellant weight and f the fraction equivalent to TNT. The fW_p lb is converted into a hemispherical shock front which propagates outward. The overpressure-distance relation as a function of TNT weight can be computed from the Kingery-BRL equation,[8] which was derived by measuring pressures from 5, 20, and 100-ton TNT explosions. We consider casualties in three categories; 2.0 psi side-on overpressure to damage eardrums for people in the open (category 1), 3.0 psi to collapse frame dwellings (category 2), and 5.0 psi to collapse or severely damage masonry buildings (category 3). The distance in feet at which a given side-on overpressure can be expected is

$$r = k(fW_p)^{1/3} \tag{5}$$

For category 1, $k = 27.5$; for category 2, $k = 20.0$; and category 3, $k = 12.0$.

Firebrand Casualties

Since fW_p pounds of propellant were converted into the pressure front, there remains $(1 - f)W$ lb§ to spread axisymmetrically about the impact point as burning propellant. Let $w_f(r)$ be the average fragment weight as a function of radius and $\eta_f(r)$ be the number of fragments per unit area. Several detailed debris maps are presented by Weals and Wilson[7] which serve to establish these functions. In these particular explosions there were numerous pieces of unburned propellant on the ground. A large number of burning pieces were also thrown out, and a still photograph taken 5 sec after one explosion shows approximately 1000 pieces in the air. This explosion had a 32% TNT equivalent. Figure 1 shows w_f, and Fig. 2 presents η_f fitted to donor test data. The four points at 750-ft radius are debris from four quadrants around the explosion point, the two points at other radii are data from diametrically located areas. There are no data

§ The figure $(1 - f)W_p$ assumes that only fW_p lb of propellant are required to give fW_p lb of TNT equivalence. Actually more than fW_p would be converted in the explosion, if TNT equivalence is based on peak pressure, and alternatively less than fW_p lb if based on impulse (solid propellants are more energetic than TNT). The value $(1 - f)W_p$ is a compromise between these two definitions.

in the 0–500 ft range; the report states the pieces were too small and numerous to count. Other debris maps show these trends, particularly the number of fragments with distance. Reference 10 states that upon impact of a Minuteman missile first stage there were 1600 burning pieces counted on a still photograph. The impact weight was approximately 32,000 lb. The deflagration was equivalent to 10% TNT, there were 28,000 lb of burning debris and an over-all average weight of 18 lb per fragment. There were many fragments hidden by smoke, so the number could easily have been twice that many giving an over-all average of 9 lb, which would fall within the range shown in Fig. 1.

Expressing conservation of propellant weight mathematically in terms of w_f and η_f we can write

$$(1 - f)W_p = \int_0^a w_f \eta_f 2\pi r dr \tag{6}$$

where 2πrdr is the elemental area of an annular region of width dr and a is the maximum throw radius. The maximum throw radius for most large missile failures is about 2500 ft.

The following functions approximately fit the data in Figs. 1 and 2:

$$w_f = \alpha r^m \tag{7}$$

$$\eta_f = A \cos^n(\pi r/2a) \tag{8}$$

where α, A are constants and m, n are positive integers. A value of $m = 2$ and $n = 4$ is plotted in Figs. 1 and 2. The amplitude A will be adjusted such that propellant weight is conserved. Inserting Eqs. (7) and (8) into (6)

$$(1 - f)W_p = 2\pi\alpha A \int_0^a r^{m+1} \cos^n \frac{\pi r}{2a}\, dr \tag{9}$$

and solving for A we obtain

$$A = (1 - f)W_p \Big/ 2\pi\alpha \int_0^a r^{m+1} \cos^n \frac{\pi r}{2a}\, dr \tag{10}$$

For large propellant weights, η_f will be large according to the last equation.

Assume that the propellant pieces are rectangular parallelepipeds with edge dimensions of d, d, $2d$. Letting ρ be the propellant density we can write

$$2\rho d^3 = w_f \tag{11}$$

and assuming the casualty area is the plan view area, $d \times 2d$, the average plan view area of pieces as a function of r becomes

$$2d^2 = 2[(\alpha/2\rho)r^m]^{2/3} \tag{12}$$

Next, let us take into account a person's dimensions by finding the radius of a circular piece of debris having the same area and add on 2 ft to obtain a new casualty area for the pieces as a function of r

$$A_c(r) = 2(1 + d)^2 \tag{13}$$

The total casualty area from the impact of a stage for people in the open is the integrated product of the average casualty area of pieces and η_f, the number of pieces per unit area

$$A_{c11} = \int_{r_1}^a [(2d)^{1/2} + (2)^{1/2}]^2 A \cos^n \frac{\pi r}{2a} \cdot 2\pi r dr \tag{14}$$

Using the relation

$$(2d^2)^{1/2} = (2)^{1/2}(\alpha/2\rho)^{1/3} r^{m/3} \tag{15}$$

and inserting this into Eq. (14)

$$A_{c11} = 4\pi A \int_{r_1}^{A} r\left[\left(\frac{\alpha}{2\rho}\right)^{2/3} r^{2/3m} + 2\left(\frac{\alpha}{2\rho}\right)^{1/3} r^{m/3} + 1\right] \cos^n \frac{\pi r}{2a}\, dr \tag{16}$$

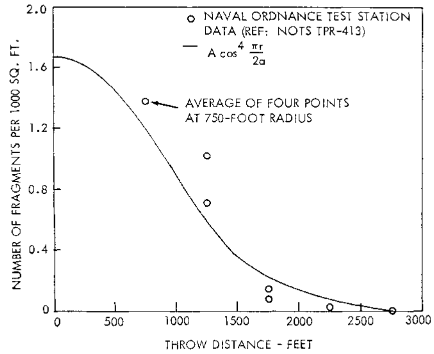

Fig. 2 Fragment density at radial locations.

The integral extends from r_1, the overpressure radius, and not zero since we have already accounted for casualties out to r_1. Equation (16) gives the casualty area for people in the open. The casualty area expression for people in light and heavy structures can be derived in a similar manner. However, the lower limits on the integrals extend from the radii at which pieces are of sufficient size to penetrate light and heavy structures.

The impacting propellant weight W_p is a function of both time to missile breakup and time to impact from breakup. Solid-propellant stages have a constant burn rate thus permitting W_p to be expressed as a linear function of flight time. Furthermore, in many cases the time to fall t_f can be expressed as a linear function of breakup time t_b; although, this depends on the particular vehicle launch trajectory. For these cases W_p can be expressed as

$$W_p = A - Bt_b \tag{17}$$

A compact expression can now be written for the casualty area resulting from overpressure and burning propellant debris for each category. For protection category 1 (people in the open) the casualty area resulting from overpressure and firebrands, has the form

$$A_{c11} = Q(A - Bt_b) + R(A - Bt_b)^{2/3} \tag{18}$$

where Q is determined by Eq. (16) and R from Eq. (5). A similar expression can be written for A_{c21} and A_{c31}, the casualty areas for light and heavy structures due to overpressure and burning debris.

The hardware debris, such as batteries, nozzles and electronic packages, must be examined to determine which pieces are expected to injure people inside light and heavy structures. Usually this requires calculating the impact velocity and estimating the penetration depth through wood or concrete. A battery may be labeled as piece 5, for example, with $A_{c15} = 7$, $A_{c25} = 7$, $A_{c35} = 0$, that is, the battery will not penetrate a concrete building but has a casualty area of 7 ft^2 to people in the open and in light frame buildings.

Numerical Example

A numerical example is presented for a hypothetical missile to show how the casualty expectation is computed. Consider a two-stage missile with a first stage propellant weight of 30,000 lb and second stage propellant weight of 10,000 lb. Assume that $f = 0.1$ for each upon impact, the missile is destroyed during first stage flight at $t_b = 10$ sec with an 8 sec free fall. Furthermore, assume the first stage nominal burn rate is 1000 lb/sec and is reduced to 200 lb/sec after destruct. Thus, 11,600 lb is consumed before impact with 28,400 lb impacting. The TNT equivalent weight is 2840 lb.

The overpressure casualty areas for categories 1–3 are 473,000, 252,000, and 90,500 ft^2, respectively. The firebrand casualty area is computed using Eq. (16). For category 1,

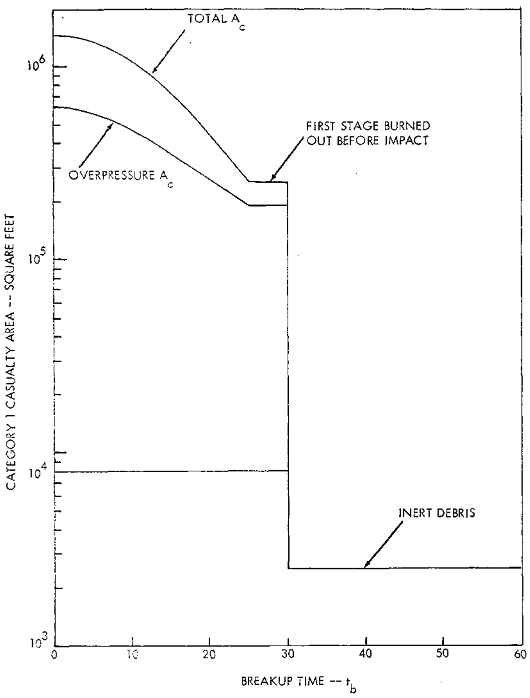

Fig. 3 Time varying casualty area.

the lower limit, r_2, is 390 ft according to Eq. (5). Assuming a maximum throw radius of 2500 ft and a propellant density of 92 lb/ft^3, the factor Q in Eq. (18) is 10.47 and the casualty area is 298,000 ft^2. This area is doubled to account for the resilient propellant which is often seen bouncing in flight failure movies. Assume a 5 lb piece is the minimum size that will penetrate a frame dwelling roof and a 15 lb piece for a masonry roof. Figure 1 shows that pieces beyond 1800 ft penetrate the frame roof and no propellant debris penetrates the masonry roof. Q for category 2 is 2.2 and the casualty area is 62,500 ft^2. Assume that the inert debris casualty areas for categories 1–3 are 8000, 5000, and 1000 ft^2, respectively. Total casualty areas for categories 1–3 are 1,077,000, 319,500, and 91,500 ft^2, respectively. The casualty area for other breakup times is shown in Fig. 3. After 30 sec the first stage is exhausted and the second stage free fall is great enough to consume the burning propellant before impact. Of course, this depends on the trajectory.

The casualty expectation is computed using Eq. (3). Assume an impact probability of 10^{-7} (these data are available for many missiles), $A_T = 27.2 \times 10^6$ ft^2 (1 square mile) and populations of 2000 in the open, 2000 in frame dwellings, and 500 in masonry structures. The casualty expectation E_c for this failure time is 0.8×10^{-5} or roughly 1 chance in 100,000 that one person will be a casualty. Kill expectations can readily be obtained by using lethal areas. The lethal area to people in the open is computed using a 10 psi side-on overpressure. A bouncing piece of propellant is not considered lethal. The lethal areas work out to be about one-tenth of the casualty areas and thus kill expectations are an order of magnitude less than casualty expectations.

Accuracy of the Hazards Model

With the number of assumptions that have been used in the development of the hazards model, the question arises as to its accuracy and utility. To answer this, return to the basic expression, Eq. (3), for the casualty expectations, but consider only the casualty area for the jth piece ($m = 1$). To separate the elements of the equation, take the logarithm,

$$\ln E_{cj} = \ln P_I + \ln N + \ln A_c - \ln A_T + \ln\left(\sum_{i=1}^{n} n_i a_{ij}\right) \quad (19)$$

Next, take the derivative, but leave the terms in the logarithmic form

$$d(\ln E_{cj}) = d(\ln P_1) + d(\ln N) + d(\ln A_c) - d(\ln A_T) + d\left[\ln\left(\sum_{i=1}^{n} n_i a_{ij}\right)\right] \quad (20)$$

Assuming these variables to be statistically independent, the variance of $\ln E_{cj}$ is

$$\sigma^2_{\ln E_{cj}} = \sigma^2_{\ln P_I} + \sigma^2_{\ln N} + \sigma^2_{\ln A_c} + \sigma^2_{\ln A_T} + \sigma^2_{\ln}\left(\sum_{i=1}^{n} n_i a_{ij}\right) \quad (21)$$

There is a convenience to the logarithmic form because estimated variability factors (EVF) can be expressed in terms of factors rather than percentages. For instance, P_I can be off by a factor of two, which means the $\pm l\sigma$ limits are equivalent to $\frac{1}{2}P_I$ and $2P_I$. The estimated variabilities of the terms in Eq. (21) are given in Table 1. Using the standard deviations from Table 1 we find that $\sigma_{\ln E_{cj}} = 1.061$, or that the variability of E_{cj} is equal to 2.89.

Noting the Central Limit Theorem,[11] and the form of Eq. (19), $\ln E_{cj}$ approaches a lognormal distribution with[12]

$$\sigma E_{cj}{}^2 = E_{cj}{}^2[\exp(2\sigma_{\ln E_{cj}{}^2}) - \exp(\sigma_{\ln E_{cj}{}^2})] \cong 5.5E_{cj}{}^2 \quad (22)$$

Next, summing over all n pieces to find $\sigma_{E_c}{}^2$,

$$\sigma^2_{E_c} = \sum_{j=1}^{n} 5.5E_{cj}{}^2 \quad (23)$$

Note that if the casualty expectation of one item of debris

Table 1 Inputs to the casualty expectation accuracy evaluation[a]

Term	Description	Reason for variability	EVF	$\sigma = \ln(\text{EVF})$
P_I	Probability of impacting in a particular inhabited area	Impact dispersion models are only approximations of the time dispersions	2	0.693
A_c	Reference casualty area (based on piece dimensions or the casualty area to people in the open exposed to an explosion)	The same generic pieces vary in size from one failure to the next. Equivalent TNT yields and propellant throw-out vary also.	2	0.693
N	Population size	Uncertainties in census	1.1	0.095
A_T	Total land area of the population center	Some population center land areas have to be estimated	1.05	0.049
$\sum_{i=1}^{n} n_i a_{ij}$	The term which assigns portions of the population (n_i) to various protection levels and accounts for the corresponding change in size of the lethal area (a_{ij})	Variations in population distribution at different times of the day, uncertainties in structure strength, uncertainties in concentration and bouncing of propellant pieces.	1.5	0.405

[a] EVF = estimated variability factor; σ = standard deviation for Eq. (21).

(the kth, for example), dominates all others, the uncertainty of E_{ck} will dominate Eq. (22) and the uncertainty in casualty expectation for all pieces will be almost equivalent to that for the kth piece alone. Assume in this case that the E_{cj}'s of three pieces, e.g., rocket motor casing and propellant parts, are approximately equal and dominate all others. Then $\sigma_{E_c} \approx 4E_{cj}$, which indicates that the standard deviation of E_c in this case is about half an order of magnitude and can range from about 2.5 to an order of magnitude.

The procedure used to arrive at the uncertainty in E_c assumed uniform P_I and did not acknowledge that the P_I from piece to piece for debris falling in a single general area is completely correlated. This omission slightly underestimates σ_{E_c} but not enough to change the range of one-half order to one order of magnitude. The method of computation of E_c is also altered when considering P_I changing vs time of failure during flight but again not change the range described.

Conclusions

The principal intent of this paper has been to present a method for estimating casualties or fatalities. This has been accomplished by the development of a model which considers the gross features of the explosion of the missile upon impact and the general protection that may be available to the exposed people. The accuracy (one standard deviation) has been found to be within an order of magnitude. Because the primary use of the model is to control the planning of missile flights in order to protect people, most assumptions tend to be conservative. Thus, the computed casualty expectations will tend to be higher than the true value. The methods and equations used are equally applicable to casualty or kill depending upon the requirements of the particular evaluation.

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